Reconstructing cosmological fields using tessellation methods

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Abstract. Astronomical observations, physical experiments as well as computer simulations often involve discrete data sets supposed to represent a fair sample of an underlying smooth and continuous field. Reconstructing the underlying fields from a set of irregularly sampled data is therefore a recurring key issue in operations on astronomical data sets. Conventional methods involve artificial filtering through a grid or a smoothing kernel and fail to achieve an optimal result. Here we describe a fully self-adaptive geometric method which does not make use of artificial filtering, and which makes optimal use of the available information.

1 The Method

Given a sample of field values at a discrete number of locations along one dimension, various prescriptions are known for reconstructing the field over the full spatial domain. In the left column of fig. 1 the familiar 0th and 1st order approaches are shown. In the right column of fig. 1 the natural generalization to two dimensions is shown. A natural choice for a 0th order multidimensional interpolation interval is the Voronoi cell centered on any of the sample points [2]. Delimiting the influence region of a point, the Voronoi cell consists of the region of space closer to the point than to any other point in the sample. The 0th order interpolation scheme sets the value of the field constant inside each Voronoi cell, its value everywhere in the cell equal to that at its nucleus (top right fig. 1). The 1st order interpolation scheme is one in which the multi-dimensional linear interpolation interval is the Delaunay cell (bottom right fig. 1). Compelling evidence for the superb performance of this interpolation scheme has been borne out by the work of Bernardeau & van de Weygaert [1]. It yields a surprisingly accurate rendering of the statistical distributions of a discretely sampled velocity field. This was particularly significant as it uncovered systematic flaws in the conventional grid-based interpolation schemes.

The implementation to any spatially discretely sampled field is rather straightforward. However, an annoying complication occurs when seeking to reconstruct the corresponding density field. The values of the density field are not a priori specified or passed along with the point sample. Instead, they have to be evaluated from the point locations themselves. Schaap & van de Weygaert [2] showed that in the case of 0th order interpolation it is indeed the inverse of the volume of the Voronoi cell that represents the appropiate density estimator. On the other hand, the simple Voronoi cell volume inverse does not suffice as density estimator in the Delaunay scheme: the resulting integrated linearly interpolated density field does not conserve mass. The solution to this problem is to use the inverse of the volume of the corresponding *contiguous* Voronoi cell, the region defined by the union of all contiguous Delaunay tetrahedra connected to a particular point.

2 Application

Cosmological N-body simulations provide an ideal template for illustrating the virtues of our method. As may be appreciated from fig. 2, they contain a large variety of objects, with diverse morphologies and a large reach of densities, spanning over a vast range of scales. The Voronoi-Delaunay method not only turns out to provide a mere improvement in density estimation, but in particular manages to deal fully self-adaptive – i.e. without any artificially manipulated filtering parameters – with two critically important properties of cosmological density fields:

- anisotropies (e.g. filaments and walls)
- hierarchies (from extended low density voids to compact, high-density clusters)

Conventional methods are usually tuned for uncovering a few aspects of the full array of properties, and may therefore be insensitive to unsuspected but intrinsically important structural elements.

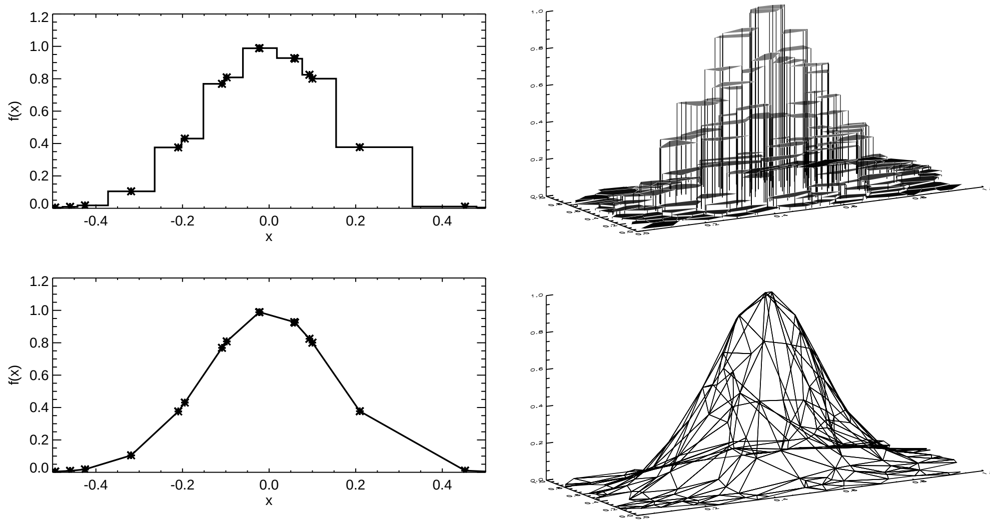

Figure 1: 0th and 1st order interpolation in one (left column) and two (right column) dimensions.

Figure 2: Figure comparing the Delaunay density estimator (central panel) with a conventional grid-based TSC method (right panel) in analyzing a cosmological N-body simulation (left panel).

The outstanding performance of our method is illustrated by fig. 2, in which its performance (central panel) on a cosmological N-body simulation is compared with that of a conventional grid-based technique (right panel). The slice depicted contains two rich clusters, some filamentary structures as well as almost empty low-density regions. Clearly the Delaunay method is able of automatically resolving all the details present in the particle distribution, while finer details are smoothed away by the grid-based method: in the TSC reconstruction the high-density clusters appear to be mere featureless blobs! In addition, low density regions are rendered as slowly varying regions at moderately low values, while the TSC reconstruction is plagued by annoying shot-noise effects. Also, our method is able to reproduce sharp, edgy and clumpy filamentary structures.

The presented example provides ample evidence of the promise of tessellation methods for the aim of continuous field reconstruction. We therefore feel that the application of tessellation methods to an array of astronomical data analysis problems is more than warranted. Indeed, it has prompted us to extend its operation to more complex, noise- and selection-ridden situations.

References

[1] Bernardeau, F., van de Weygaert, R., 1996, MNRAS 279, 693

[2] Schaap, W.E., van de Weygaert, R., 2000, A&A 363, L29; 2001, in preparation